Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

ARTICLE TYPE

Bearing-Based Formation Stabilization Using Event-Triggered Control

Mayank Sewlia[1] | Daniel Zelazo[2]

[1]Division of Decision and Control Systems, KTH Royal Institute of Technology, Stockholm, Sweden
[2]Faculty of Aerospace Engineering, Technion-Israel Institute of Technology, Haifa, Israel

Correspondence
Daniel Zelazo. Email: dzelazo@technion.ac.il

Summary

This paper studies two distributed bearing-based event-triggered schemes to achieve formation stabilization. We focus on systems with double-integrator dynamics with bearings sensing capabilities. First, we propose a bearing-only event-triggered condition (ETC) that is edge-dependent which drives the control updates of the agents using only information dependent on relative sensed quantities. Second, along with bearing measurements we make use of local agent state measurements to arrive at an ETC that uses this collective measurement to drive the sensing and control updates of an agent. In doing so, we propose a new control law that renders the final formation stationary. Simulations are provided to verify the validity of the proposed algorithms.

KEYWORDS:
formation control, multi-agent system, event-triggered control

1 | INTRODUCTION

Multi-agent systems (MAS) are systems composed of multiple interacting dynamic units that work cooperatively with each other to achieve a common goal. They are often characterized by autonomy (agents responsible for their own actions), adaptability (respond to changing environment), complexity (resulting from decision-making using locally available information) and distributed architectures [1]. With an increase in processing power, MAS have been recently assigned to solve a plethora of problems such as flocking, formation control, and distributed estimation. These problems are often accompanied with their own set of challenges such as large-scale implementation, stability, and communication overload [2].

Formation control problems [3], one of the most studied problems in the field of MAS, aim to achieve a target formation shape by defining constraints on the relative states of the agents. Types of formation control problem includes distance-based, displacement-based, shape-based and bearing-based formations. In bearing-based formation control problems, the target formation is specified by inter-agent bearings and the sensing measurements are relative bearing vectors. As noted in the literature, bearing measurements are often cheaper to sense and a formation specified by bearings is invariant to translation and scaling, thereby providing easy maneuverability in these aspects [4,5]. Most recently, several continuous time bearing-only control laws for varied agent models including single-integrator, double-integrator and unicycle dynamics were proposed [6].

In the existing literature, the assumption of very high sampling frequency and actuation rate is very common. This is difficult to maintain as real world implementations on digital beds have limited communication and processor capabilities [7]. Additionally, once the update rate crosses a certain threshold, there is no improvement in the final convergence accuracy. As a result, we need to find a suitable update frequency while still achieving a desired accuracy. Moreover, this update frequency also depends on the current states of the system and varies throughout the evolution of the agents. Hence, it is intuitive to sample and update states at higher frequencies only when the situation demands, and sample at lower frequencies otherwise. To solve this problem, we make use of event-triggered control (ETC) where we opportunistically and deliberately find sampling and update instants without compromising the final results[8]. An extensive review is presented on the topic of event-triggered control studying the motivation, methodology, challenges and applications of such a control in distributed systems[9].

There has been little work in applying event-triggered control to solve formation control problems. Notably, Yu et. al.[10], studies bearing-based encirclement formation control using event-triggered schemes for single integrator dynamics. The authors demonstrate global asymptotic stability and prove the avoidance of zeno behavior. However, the event triggering is centralized in the sense that all agents sample and perform control updates at the same instance. The work[11] provides dynamic event-triggered conditions to solve the formation control problem for agents with linear dynamics. The ETC condition is dynamic in the sense that the event variable 'σ' is gradually reduced closer to 0 from a certain initial value as time progresses. Here, the sensing variables are the states of neighboring agents along with bearing vectors in a global reference frame. Obtaining both states and bearings of neighbouring agents require dedicated bearing sensors and a direct communication channel to exchange state information which can be too demanding. Additionally in[12], position-dependent event-triggered formation control problem for agents with single integrator dynamics is studied and the authors propose an algorithm to solve the multi-target selection problem. A generalized gradient based control law is proposed in[13] for agents with single-integrator dynamics and exponential convergence is shown for this controller. Here, the ETC is a function of the smallest eigenvalue of a matrix which is a property of the graph. This is sometimes undesired as it is a global property of the network, and can not cope with communication or agent losses. On the other hand, the authors in[14] propose edge-based event-triggered control without use of global information but are limited in the types of achievable desired formation. More recently[15] proposes multiple average consensus algorithms for discrete-time event-triggered systems. Leader-follower flocking[16] of second-order multi-agent system with event-triggered control is studied, however, agents continuously broadcast their positions and use ETC to broadcast individual velocities. We avoid this by having ETC drive both communication and control updates.

This work varies from the previous works as follows: Firstly, we address the problem for agents with second-order dynamics and the proposed control inputs along with the ETC are distributed. Secondly, we design event triggered conditions to achieve formation stabilization. The latter part is solved using two different scenarios, namely, *edge triggering* and *node triggering*. In edge triggering, we make use of the bearing-only control law proposed in[6] to arrive at a bearing-based ETC that is dependent on the relative measurements applied to both agents incident to that edge.[1] Here, the post-trigger response is to update the controller when the ETC is violated. In edge triggering, the event-triggered condition is defined on information over each communication link between a pair of agents. If this condition is violated, the agents incident to such an edge execute the post-trigger response simultaneously. In traditional node triggering, if an event-triggered condition is violated for agent i, then agent i along with all its neighbors perform control action synchronously. In node triggering, we first design a control law to achieve global asymptotic stability where the desired goal is to achieve a stationary end-formation. Then we use this controller to derive an ETC. Here the ETC is designed over an agent, that along with its self-states uses the collective event-triggered relative measurements from all its neighbors to evaluate the trigger condition. The post-trigger response in this case is to acquire new state measurements and update the controller. Our current focus is solely on the ideal dynamics of the system, without taking into account uncertainties and disturbances. Despite the fact that these factors can significantly impact the system's performance, the primary objective of this work is to investigate how event-triggered conditions are utilized in bearing-based systems to reduce communication and control costs.

The rest of the paper is organized as follows. In Section 2 we discuss some relevant preliminaries along with the system model and pose the problem we are trying to solve. In Section 3, two main theorems are presented which develop the required ETCs using Lyapunov theory. In Section 4 we show an example to demonstrate the effectiveness of the proposed results and finally conclude the paper by presenting some remarks in Section 5.

Notations:

Consider n mobile agents and let their positions be represented by $p_i(t) \in \mathbb{R}^d$ and their velocity by $v_i(t) \in \mathbb{R}^d$. Communication between the agents is given by a fixed and connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consisting of a finite vertex set $\mathcal{V} = \{a_1, a_2, \ldots, a_n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ with $n = |\mathcal{V}|$ and $m = |\mathcal{E}|$. The set of neighbors of vertex i is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$. To each edge in $\mathcal{G}$, assign a label $q \in \{1, \ldots, m\}$ and an arbitrary direction. The incidence matrix $H \in \mathbb{R}^{n \times m}$ is defined element wise as, $H_{iq} = -1$ if vertex i is the tail of edge q and $H_{iq} = 1$ if vertex i is the head of edge q and $H_{iq} = 0$ otherwise. We denote $\bar{H} = H \otimes I_d$, where $\otimes$ is Kronecker product.

[1] An edge between two agents indicates that a bearing measurement is available.

2 | PRELIMINARIES AND PROBLEM STATEMENT

In this section, we give an overview of the bearing-only formation control problem and present the system model to be studied. Then we pose the problem we aim to solve by giving some basics on event-triggered control and present an outline of our methodology.

2.1 | Bearing-Based Formation Control

In formation control problems, the aim is to drive an ensemble of agents to a desired geometric pattern determined by constraints on their states. These constraints can be specified using relative states between the agents, as is the case in bearing-based formation control[4]. Here, the constraints are defined in the form of inter-agent bearings and the formation is achieved when the measured bearing vector g_{ij}, expressed in a global coordinate frame, arrives at a desired bearing vector $g^{\star}_{ij}$. Some of the tools required to facilitate our analysis are defined below.

We start by defining an orthogonal projection matrix operator $P : \mathbb{R}^d \to \mathbb{R}^{d\times d}$ as, $P(x) = P_x \triangleq I_d - (xx^T)/\|x\|^2$, where $x \in \mathbb{R}^d$ is a nonzero vector and $d \geq 2$. The matrix P_x is positive-semi definite with one zero eigenvalue and $d-1$ eigenvalues at 1. A framework in $\mathbb{R}^d$ is a pair $(\mathcal{G}, p)$ that maps every vertex in the graph $\mathcal{G}$ to a point in the space $\mathbb{R}^d$. The relative position and velocity vectors between two agents (the *edge* vectors) are defined as, $e_{ij} \triangleq p_j - p_i, \quad \dot{e}_{ij} \triangleq v_j - v_i, \quad (i,j) \in \mathcal{E}$. The *bearing* and *rate of bearing* vector between two agents are defined as,

$$g_{ij} \triangleq \frac{e_{ij}}{\|e_{ij}\|}, \, (i,j) \in \mathcal{E}$$
$$\dot{g}_{ij} \triangleq \frac{P_{g_{ij}}}{\|e_{ij}\|}\dot{e}_{ij}, \, (i,j) \in \mathcal{E},$$

as illustrated in Figure 1. The vectors e and $\dot{e}$ can thus be expressed in compact form as $e = \bar{H}^T p$ and $\dot{e} = \bar{H}^T v$. Since $P_{g_{ij}} g_{ij} = 0$, $g^T_{ij}\dot{g}_{ij} = e^T_{ij}\dot{g}_{ij} = 0$. The set of desired bearings are denoted by $\{g^{\star}_{ij}\}_{(i,j)\in\mathcal{E}}$. All of these quantities are expressed in a global coordinate frame. This, for example, allows one to write $g_{ij} = -g_{ji}$.

Definition 1 (Feasible Formation)**.** A target formation is said to be *feasible* if there exists at least one configuration p that satisfies the bearing constraints, that is $g_{ij} = g^{\star}_{ij}$ for all $(i,j) \in \mathcal{E}$.

However, feasibility of bearing constraints does not imply uniqueness of the target formation. To study the existence of unique formations, denote the bearings of a directed edge as g_k where $k \in \{1, \dots, m\}$ and define the bearing function $F_B : \mathbb{R}^{dn} \to \mathbb{R}^{dm}$

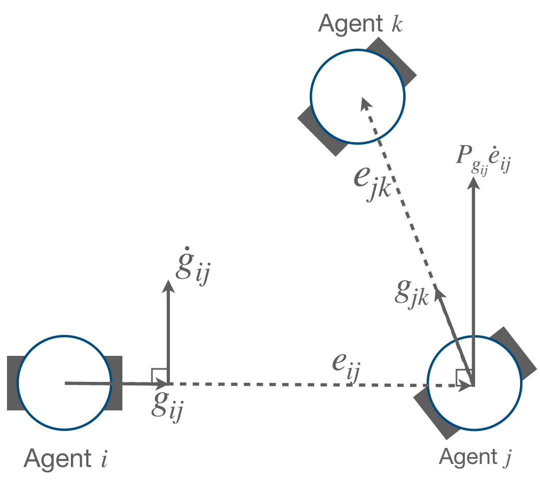


FIGURE 1 Geometric relation between relative vectors.

as, $F_B(p) \triangleq \left[g_1^T, \dots, g_m^T\right]^T$. The *bearing rigidity matrix* is defined as the Jacobian of the bearing function,

$$R_B(p) \triangleq \frac{\partial F_B(p)}{\partial p} \in \mathbb{R}^{dm \times dn}.$$

Let δp be a variation of p such that if $R_B(p)\delta p = 0$ then δp is called an infinitesimal bearing motion. There are two kinds of trivial infinitesimal bearing motions: translation and scaling, which leads us to our next definition,

Definition 2 (Infinitesimal Bearing Rigidity)**.** A framework is *infinitesimally bearing rigid* if all the infinitesimal bearing motions are trivial.

A framework which is infinitesimally bearing rigid can be uniquely defined up to a translation and scaling factor[4].

Assumption 1. Assume the target formation specified by the desired bearings $\{g_{ij}^\star\}_{(i,j)\in\mathcal{E}}$ is feasible, i.e., there exists a configuration p that satisfies $g_{ij} = g_{ij}^\star$ for all $(i,j) \in \mathcal{E}$. Assume further that at this configuration the framework is infinitesimally bearing rigid.

The existence and analysis of unique target formation in bearing-based control comes under the study of *bearing rigidity theory*; see[5]. In this work, we consider multi-agent systems having double integrator dynamics described as,

$$\dot{p}_i(t) = v_i(t),\ \ \dot{v}_i(t) = u_i(t) \quad i = 1, \dots, n, \tag{1}$$

where $p_i(t) \in \mathbb{R}^d$, $v_i(t) \in \mathbb{R}^d$ and $u_i(t) \in \mathbb{R}^d$ are position, velocity and control inputs of agent i, respectively. While working with double integrator models, it is not sufficient to only sense the relative bearing vectors, as we also need some information on the relative velocities of the agents. This information, in some part, is provided by sensing the rate of bearings $\{\dot{g}_{ij}\}_{(i,j)\in\mathcal{E}}$. Although the velocities of agents need not synchronize, the rate of bearings do i.e., $\{\dot{g}_{ij}\}_{j\in\mathcal{N}_i} \to 0$ as $t \to \infty$. Consider the bearing-only control law presented in[6] for the system defined in (1) as,

$$u_i(t) = k_p \sum_{j\in\mathcal{N}_i} (g_{ij}(t) - g_{ij}^\star) + k_v \sum_{j\in\mathcal{N}_i} \dot{g}_{ij}(t), \tag{2}$$

where k_p and k_v are positive position and velocity gains respectively, $\{g_{ij}\}_{(i,j)\in\mathcal{E}}$ are the relative bearings and $\{\dot{g}_{ij}\}_{(i,j)\in\mathcal{E}}$ are the rate of bearings. The closed-loop system is,

$$\dot{v}(t) = -k_p \bar{H}(g(t) - g^\star) - k_v \bar{H}\dot{g}(t). \tag{3}$$

As seen in[6], the system in (3) has an equilibrium when the final formation moves at a constant velocity.

However, since the focus of the latter half of this paper is to find an ETC for a stationary final formation, we propose the modified control law,

$$u_i(t) = k_p \sum_{j\in\mathcal{N}_i} (g_{ij}(t) - g_{ij}^\star) + k_v \sum_{j\in\mathcal{N}_i} \dot{g}_{ij}(t) - k_v v_i(t), \tag{4}$$

where $v_i \in \mathbb{R}^d$ is the velocity of agent i. In matrix form, the control law (4) is,

$$u(t) = -k_p \bar{H}(g(t) - g^\star) - k_v \bar{H}\dot{g}(t) - k_v v(t). \tag{5}$$

Before we provide the stability and convergence proof of the proposed controller (5), we introduce a lemma from[6]:

Lemma 1 ([6])**.** Suppose none of the agents coincide, i.e., $p_i \neq p_j\ \forall\, i,j \in \mathcal{V}$, then $p^T \bar{H}(g(t) - g^\star) \geq 0$ and equality exists if and only if $g(t) = g^\star$.

It is important to note that the bearing measurements (and their derivatives) are not well-defined when agents collide. It is therefore common in the literature to assume collisions do not occur[17,4], and we introduce this assumption below. It is possible to derive sufficient conditions for the initial conditions that ensure no collisions[4], but this is beyond the scope of this work.

Assumption 2. There are no agent collisions with each other during the evolution of the formation dynamics.

Theorem 2. Suppose Assumption 1 and 2 and Lemma 1 hold. Then, the control law given by (4) induces asymptotic convergence of the system state $g(t)$ to the target configuration $g^\star(t)$, where $g^\star(t)$ represents the target configuration moving at zero velocity.

Proof. Define $V : \mathbb{R}^{2nd} \to \mathbb{R}$, a continuously differentiable function as, $V(p, v) = k_p p^T \bar{H}(g - g^\star) + \frac{1}{2} v^T v$. Using Lemma 1 and $v^T v > 0\ \forall\ v \in \mathbb{R}^{nd} - \{0_{nd\times 1}\}$, we note that $V(p, v)$ is positive definite. It's time derivative is,

$$\dot{V}(p, v) = k_p(g - g^\star)^T \bar{H}^T v + v^T \dot{v}, = -k_v v^T \bar{H}\dot{g} - k_v v^T v.$$
$$= -k_v v^T \Big(\bar{H}\mathrm{diag}\Big(\frac{P_{g_{ij}}}{\|e_{ij}\|}\Big)\bar{H}^T + I\Big) v < 0, \tag{6}$$

where the last inequality is due to $\mathrm{diag}\Big(\frac{P_{g_{ij}}}{\|e_{ij}\|}\Big)$ being positive-semi definite and hence $\Big(\bar{H}\mathrm{diag}\Big(\frac{P_{g_{ij}}}{\|e_{ij}\|}\Big)\bar{H}^T + I\Big)$ is positive definite. With the above Lyapunov function, we define a compact set $\Omega_a = \{(p, v) \in \mathbb{R}^{2nd} | V(p, v) \leq const\}$ and let $S = \{(p, v) \in \Omega_a | \dot{V} = 0\}$. From (6) we note that $\dot{V} = 0$ if and only if $v(t) = 0$ which implies the formation is stationary and hence $\dot{g}(t) = 0$. Additionally, $v(t) = 0 \Rightarrow \dot{v}(t) = 0$ and from (5) we obtain $-k_p \bar{H}(g(t) - g^\star) = 0$. Left multiplying this expression by p^T, we arrive at the result of Lemma 1 which is true if and only if $g(t) = g^\star$. The proof is complete by invoking Lyapunov's stability theorem. □

2.2 | Event-Triggered Control

The general notion of event-triggered control, as presented in[9], is to define an event-triggered condition that executes a trigger response when a locally computed error function exceeds a state-dependent threshold. Such functions generally take the form, $f(x, e) \triangleq m(e) - n(x)$, where $m(e)$ is some function of the error and $n(x)$ is a function of the states of the system. The error term is defined as the difference between the last measured state during an event time and the current state, i.e., $e = \hat{x}(t'_k) - x(t)$ where the states $\hat{x}(t'_k)$ exist during the event times: $t'_1, t'_2, \dots$ i.e., at every instance when $f(x, e) = 0$. The threshold function $n(x)$ provides bounds on the evolution of the errors, and when an event is triggered the error function is reset to zero and the states are updated as $\hat{x}(t'_k) = x(t)$. To design such functions we make use of Lyapunov theory that provides bounds on the trajectory of the system while maintaining stability by confining the trajectory to a ball around the equilibrium.

2.3 | Problem Statement

The control laws (2) and (4) are continuous time controllers with continuous bearing states $\{g_{ij}(t), \dot{g}_{ij}(t)\}$. To design an event triggered scheme we need to define discontinuous states that exist only during the event times. These states are denoted by $\{\hat{g}_{ij}(t_i^k), \hat{\dot{g}}_{ij}(t_i^k)\}$ where t_i^k is the time when an event is triggered for agent i. The problem we study is when to use these discontinuous states and solve the bearing-based formation control problem. We divide this problem into two sub-problems. In the first sub-problem we achieve bearing-based stabilization where the final formation moves with a constant velocity. In the second sub-problem we achieve bearing-based stabilization where the final formation is stationary. Formally, these two problems can be posed as follows:

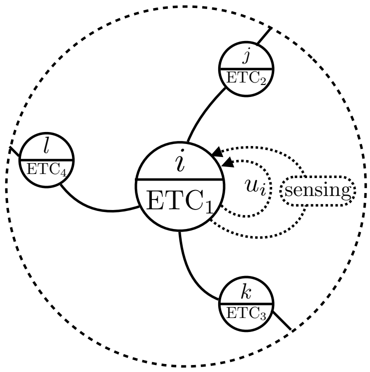
Problem 1 (Node Triggering)**.** Design an event-triggered condition for each agent $i \in \mathcal{V}$, that drives both the sensing and the control updates using the measured self states $\{p_i, v_i\}$ and the bearing states $\{g_{ij}, \dot{g}_{ij}\}$ such that $v_i \to 0\ \forall\ i \in \mathcal{V}$, $g_{ij} \to g_{ij}^\star$ and $\dot{g}_{ij} \to 0$ for all $(i, j) \in \mathcal{E}$ as $t \to \infty$.

Problem 2 (Edge Triggering)**.** Assume continuous sensing between agents, design an event-triggered condition over each edge $(i, j) \in \mathcal{E}$ that drives the control updates using only the measured bearing states $\{g_{ij}, \dot{g}_{ij}\}$ and the relative velocity $\dot{e}_{ij}$ such that $g_{ij} \to g_{ij}^\star$ and $\dot{g}_{ij} \to 0$ for all $(i, j) \in \mathcal{E}$ as $t \to \infty$.

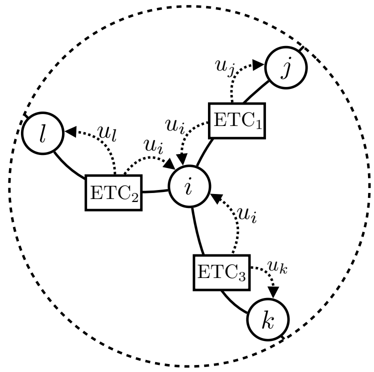
In Problem 2, we make use of the double-integrator control law (2) to arrive at a relative bearing-based ETC that is edge dependent, and in Problem 1, along with the bearing measurements we use agents self states (p_i, v_i) in the double-integrator control law (4) to arrive at an ETC which is node dependent and will drive the agents to a stationary formation.

3 | EVENT TRIGGERED CONDITION DESIGN

In this section, we design ETC's for two cases: node triggering and edge triggering. In the first case, as seen in Figure 2a, the ETC is designed over each node and every agent along with the bearing measurements also measures its self states and feeds this into the ETC. When this ETC is violated, the agent acquires new state measurements from its sensors, updates its controller and sets the error function to zero. Whereas in the second case, as portrayed in Figure 2b, the ETC is designed over each edge and every edge has its independent trigger condition. Over an edge $(i, j) \in \mathcal{E}$ agent i and agent j monitor the bearing states continuously, and when the trigger condition is violated both the agents simultaneously update their respective controllers and sets the error function to zero. The total number of event-triggered conditions to be evaluated in node triggering is $|\mathcal{V}| = n$, whereas in edge triggering $2|\mathcal{E}|$ conditions must be evaluated. To summarize, ETC drives control updates in edge triggering and ETC drives both sensing and control updates in node triggering.



(a) Node triggering.



(b) Edge triggering.

FIGURE 2 Illustration of edge and node triggering ETC.

3.1 | Node triggering

In node triggering, unlike the edge triggering case, the sensing is discontinuous and driven by the ETC which is defined over the collective information acquired from all the neighbors of agent i. Here, the trigger response is not only to update the controller but also to drive the sensing of the neighboring states. The control law (4) is different from the one proposed in (2) as it uses self velocity of the agents to achieve a stationary final formation.

The error dynamics in this case are defined with respect to the position and velocity of the agents and are as follows,

$$r_{p_i}(t) \triangleq \hat{p}_i(t_i^k) - p_i(t) \quad \& \quad r_{v_i}(t) \triangleq \hat{v}_i(t_i^k) - v_i(t), \tag{7}$$

where $t \in [t_i^k, t_i^{k+1})$. The continuous controller in (4) is modified to the piecewise continuous controller,

$$u_i(t_i^k) = k_p \sum_{j\in\mathcal{N}_i} (\hat{g}_{ij}(t_i^k) - g_{ij}^\star) + k_v \sum_{j\in\mathcal{N}_i} \hat{g}_{ij}(t_i^k) - k_v\hat{v}_i(t_i^k). \tag{8}$$

The following theorem provides an ETC and presents convergence analysis for the control law (8):

Theorem 3. Consider the system in (1) with control law (8), assume the communication graph is undirected and connected, and that Assumptions 1 and 2 hold. Suppose the ETC is given by,

$$\|r_{v_i}\|^2 > \psi(t), \tag{9}$$

where

$$\psi(t) = \frac{2\epsilon}{k_v}\left(\sigma\kappa\|v_i\|^2 - \frac{k_p}{2\epsilon}\|\sum_{j\in\mathcal{N}_i}(\hat{g}_{ij} - g_{ij}^\star)\|^2 - \frac{k_v}{2\epsilon}\|\sum_{j\in\mathcal{N}_i}\hat{g}_{ij}\|^2\right), \tag{10}$$

with r_{v_i} the sensing error in velocity as defined in (7), $\epsilon > 0$, and $\kappa = k_v(1-\epsilon) - \frac{k_p\epsilon}{2}$. Then for $\sigma \in (0, 1)$, the above ETC decides when to generate the control input $u_i(t_i^k)$ and when to measure the self- and bearing-states such that $\hat{v}_i(t_i^k) \to 0$, $\hat{g}_{ij}(t_i^k) \to g_{ij}^\star$, and $\dot{\hat{g}}_{ij}(t_i^k) \to 0$, where $g_{ij}^\star$ is the final bearing requirement to achieve the desired formation. Furthermore, the inter-event times $(t_i^{k+1} - t_i^k)$ are lower bounded by a Δ given by,

$$\Delta = (1/\alpha)\log\left(1 + \frac{\alpha^2\psi(t)}{\|u_i(t_i^k)\|^2}\right), \tag{11}$$

when $\psi > -\|u_i(t_i^k)\|\alpha^2$ where $u_i(t_i^k) \neq 0$ and $\alpha > 0$.

Proof. Define $V : \mathbb{R}^{2nd} \to \mathbb{R}$ as $V(p_i, v_i) = \sum_{i=1}^{n} \frac{1}{2} v_i^T v_i$. The function is a continuously differentiable function on the set $\Omega_c = \{(p, v) \in \mathbb{R}^{2nd} | V(p, v) \leq const.\}$ which is positively invariant to (1). Its time derivative is,

$$\dot{V}(p_i, v_i) = \sum_{i=1}^{n} v_i^T \dot{v}_i = \sum_{i=1}^{n} v_i^T \Big(k_p \sum_{j\in\mathcal{N}_i} (\hat{g}_{ij} - g_{ij}^\star) + k_v \sum_{j\in\mathcal{N}_i} \hat{\dot{g}}_{ij} - k_v(v_i + r_{v_i}) \Big).$$

Using the error definitions of (7) and Young's inequality for inner products[2], we bound the above equation as,

$$\dot{V}(p_i, v_i) \leq \sum_{i=1}^{n} \Big[k_p \Big(\frac{\epsilon \|v_i\|^2}{2} + \frac{\| \sum_{j\in\mathcal{N}_i} (\hat{g}_{ij} - g_{ij}^\star)\|^2}{2\epsilon} \Big) + k_v \Big(\frac{\epsilon\|v_i\|^2}{2} + \frac{\| \sum_{j\in\mathcal{N}_i} \hat{\dot{g}}_{ij}\|^2}{2\epsilon} \Big) - k_v\|v_i\|^2 + k_v \Big(\frac{\epsilon\|v_i\|^2}{2} + \frac{\|r_{v_i}\|^2}{2\epsilon} \Big) \Big],$$
$$= \sum_{i=1}^{n} \Big[- \Big(k_v(1-\epsilon) - \frac{k_p\epsilon}{2} \Big) \|v_i\|^2 + \frac{k_p}{2\epsilon} \| \sum_{j\in\mathcal{N}_i} (\hat{g}_{ij} - g_{ij}^\star)\|^2 + \frac{k_v}{2\epsilon} \| \sum_{j\in\mathcal{N}_i} (\hat{\dot{g}}_{ij}\|^2 \Big] + \sum_{i=1}^{n} \frac{k_v}{2\epsilon} \|r_{v_i}\|^2.$$

Restricting $\|r_{v_i}\|^2$ above to,

$$\|r_{v_i}\|^2 \leq \psi(t), \tag{12}$$

with $\psi(t)$ given in (10), results in,

$$\dot{V}(p, v) \leq \sum_{i=1}^{n} (\sigma - 1)\kappa k_v \|v_i\|^2,$$

which is negative semi-definite for $\sigma \in (0, 1)$ and $\kappa = k_v(1-\epsilon) - \frac{k_p\epsilon}{2} > 0$. Invoking the invariance principle,[18], define $S_2 = \{(p, v) \in \mathbb{R}^{2nd} | \dot{V} = 0\}$. Then $\dot{V} = 0$ implies $v_i = 0$, which implies $\dot{e}_{ij} = 0$ and $\dot{g}_{ij} = 0$. From (8) we arrive at $k_p \sum_{j\in\mathcal{N}_i} (\hat{g}_{ij} - g_{ij}^\star) = 0$. This is satisfied only when $\hat{g}_{ij} = g_{ij}^\star$ which follows from Lemma 1. This means that if the sufficient condition $\|r_{v_i}\|^2 \leq \psi(t)$ is met, then we can be sure that we will reach the desired formation. We can use this condition as a trigger, so whenever the inequality is violated, we can update our approach to maintain the guarantee of convergence.

To show the lower bound on inter-event times, we follow the proof shown in[19]. We bound the derivative of the error function as follows,

$$\frac{\mathrm{d}}{\mathrm{d}t} r_{v_i}^T r_{v_i} = -2 r_{v_i}^T \dot{v}_i \leq \Big(\alpha \|r_{v_i}\|^2 + (1/\alpha)\|u_i\|^2 \Big),$$

where we used the Young's inequality for inner products. Denoting $y = \|r_{v_i}\|^2$, we obtain $\dot{y} \leq \alpha y + (1/\alpha)\|u_i\|^2$ and y satisfies $y(t) \leq \phi(t)$ where $\phi(t)$ is the solution of $\dot{\phi} - \alpha\phi = (1/\alpha)\|u_i\|^2$. The solution to this equation is $\phi(t) = (1/\alpha^2)\|u_i\|^2(\exp(\alpha(t - t_i^k)) - 1)$ where $t \in [t_i^k, t_{i+1}^k)$. Now using the inequality (9), we arrive at the following bound for inter-event times,

$$t - t_i^k \geq (1/\alpha) \log \left(1 + \frac{\alpha^2 \psi(t)}{\|u_i(t_i^k)\|^2} \right),$$

where $\psi(t)$ is given in (10). If $\psi(t) = 0$, then condition (9) is not violated and no event is generated, hence we have $t > t_i^k$ i.e., the next event time is strictly larger than the previous event time and the inter-event time is non-zero. □

Remark 1. Note that the inter-event times in Theorem 3 are expressed in terms of the signals $\psi(t)$ and $u_i(t_i^k)$. In fact, it is straightforward to show that both $\psi(t)$ and $u_i(t_i^k)$ can be uniformly bounded by positive constants.

Applying the triangle inequality to (8), we can bound the controller as follows:

$$\|u_i(t_i^k)\| \leq \Big\| k_p \sum_{j\in\mathcal{N}_i} (\hat{g}_{ij}(t_i^k) - g_{ij}^\star) \Big\| + \Big\| k_v \sum_{j\in\mathcal{N}_i} \hat{\dot{g}}_{ij}(t_i^k) \Big\| + \Big\| k_v \hat{v}_i(t_i^k) \Big\|. \tag{13}$$

We will now individually upper bound the three terms on the right-hand side above. Recalling that the bearings are a unit vector, we see,

$$\Big\| k_p \sum_{j\in\mathcal{N}_i} (\hat{g}_{ij}(t_i^k) - g_{ij}^\star) \Big\| \leq k_p \sum_{j\in\mathcal{N}_i} \Big(\|\hat{g}_{ij}(t_i^k)\| + \|g_{ij}^\star\| \Big) = 2 k_p d_i,$$

[2] Young's inequality states that for vectors $a, b \in \mathbb{R}^n$, $a^T b \leq \frac{\|a\|^2}{2\epsilon} + \frac{\epsilon\|b\|^2}{2}$ for any $\epsilon > 0$.

where d_i is the number of neighbours of agent i. Next, recalling from Theorem 3, it follows that $\|k_v \hat{v}_i(t_i^k)\| \leq k_v \|\hat{v}_i(0)\|$. Finally, since the projection operator is non-expansive it follows that

$$\left\| k_v \sum_{j\in\mathcal{N}_i} \frac{P_{g_{ij}}}{\|e_{ij}(t_i^k)\|} \dot{e}_{ij}(t_i^k) \right\| \leq \left\| k_v \sum_{j\in\mathcal{N}_i} \frac{\dot{e}_{ij}(t_i^k)}{\|e_{ij}(t_i^k)\|} \right\|.$$

Let the norm $\|v_j(t_j^k)\|$ be bounded by $\|v_j(0)\|$ and similarly $\|v_i(t_i^k)\|$ be bounded by $\|v_i(0)\|$, then $\|\dot{e}_{ij}(t_i^k)\| \leq \|v_j(0)\| + \|v_i(0)\| \leq C_{1j}$ where C_{1j} is some positive constant. Since the velocities v_i are bounded and converge to zero, we have $\|e_{ij}(t_i^k)\| = \|p_j(t_i^k) - p_i(t_i^k)\| \leq C_{2j}$ where C_{2j} is some positive constant. Thus we obtain,

$$\left\| k_v \sum_{j\in\mathcal{N}_i} \frac{\dot{e}_{ij}(t_i^k)}{\|e_{ij}(t_i^k)\|} \right\| \leq k_v C$$

where $C = \sum_{j\in\mathcal{N}_i} \frac{C_{1j}}{C_{2j}} > 0$. We are now prepared to bound the controller from (13),

$$\|u_i(t_i^k)\| \leq 2k_p d_i + k_v \|\hat{v}_i(0)\| + k_v C, \tag{14}$$

where we assumed that no collisions take place due to Assumption 2. Therefore, $u_i(t_i^k)$ is uniformally bounded. A similar argument can be made for $\psi(t)$.

Remark 2. While providing a lower bound on the inter-event times does not exclude Zeno behavior, we note that in numerous simulations we do not observe this phenomenon. Proving the exclusion of Zeno behavior is the subject of future work.

3.2 | Edge Triggering

In traditional event-triggered schemes, agent i monitors the states of its neighbors and uses this collective information to evaluate the ETC. However, in edge triggering, we define the error dynamics on the relative bearings and rate of bearings. This means that execution of a trigger response is no longer dependent on the states of all the neighbors but on each edge. Edge $(i, j) \in \mathcal{E}$ has its trigger condition evaluated by agent i and agent j and if violated, the bearings and rate of bearings of that particular edge is updated and a new control input is generated. Assuming perfect information, agent i and agent j trigger events at the same time.

The aim of designing an ETC is to derive a threshold function which dictates how far the error function can be allowed to rise before requiring an update, i.e., how far the states of the system can evolve without updating the controller. These updates occur at event times denoted by $t_i^1, t_i^2, \dots$, and the control input between events is held constant in a zero-order hold fashion. The bearing states that exist at event times are denoted by $\{\hat{g}_{ij}, \hat{\dot{g}}_{ij}\}$ and are constant between the interval $[t_i^k, t_i^{k+1})$. To check how far the bearing states have evolved in this interval, we define time-dependent error terms $r_{g_{ij}}(t)$ and $r_{\dot{g}_{ij}}(t)$. Using this information, the relative errors are as follows,

$$r_{g_{ij}}(t) \triangleq \hat{g}_{ij}(t_i^k) - g_{ij}(t) \;\&\; r_{\dot{g}_{ij}}(t) \triangleq \hat{\dot{g}}_{ij}(t_i^k) - \dot{g}_{ij}(t), \tag{15}$$

where $t \in [t_i^k, t_i^{k+1})$. Using the above notations, we can modify the continuous controller (2) to a piecewise continuous controller by replacing the states $\{g_{ij}, \dot{g}_{ij}\}$ with $\{\hat{g}_{ij}, \hat{\dot{g}}_{ij}\}$ and the resulting closed-loop dynamics are,

$$u_i(t_i^k) = k_p \sum_{j\in\mathcal{N}_i} (\hat{g}_{ij}(t_i^k) - g_{ij}^\star) + k_v \sum_{j\in\mathcal{N}_i} \hat{\dot{g}}_{ij}(t_i^k). \tag{16}$$

The following result provides convergence analysis for the system in (1) and control law (16):

Theorem 4. Consider the system in (1) with control law (16) under Assumption 2, and further assume the communication graph is undirected and connected and that Assumption 1 holds. Suppose the event-triggered condition is given by,

$$k_p \|\dot{e}_{ij}\| \|r_{g_{ij}}\| + k_v \|\dot{e}_{ij}\| \|r_{\dot{g}_{ij}}\| > \sigma k_v \langle \dot{e}_{ij}, \dot{g}_{ij} \rangle, \tag{17}$$

where $r_{g_{ij}}$ is the sensing error in $\{g_{ij}\}_{j\in\mathcal{N}_i}$ and $r_{\dot{g}_{ij}}$ is the sensing error in $\{\dot{g}_{ij}\}_{j\in\mathcal{N}_i}$ as defined in (15). Then the ETC in (17) decides when to generate the control update $u_i(t_i^k)$ such that $\dot{g}_{ij}(t) \to 0$ and $g_{ij}(t) \to g_{ij}^\star$, where $g_{ij}^\star$ the desired bearing constraint.

Proof. Define $V : \mathbb{R}^{2nd} \to \mathbb{R}$, a continuously differentiable function on the set $\Omega_b = \{(p, v) \in \mathbb{R}^{2nd} | V(p, v) \leq const.\}$ which is positively invariant to (1) as $V(p, v) = k_p e^T (g - g^\star) + \frac{1}{2} v^T v$. It's time derivative is,

$$\begin{aligned}
\dot{V}(p, v) &= k_p (g - g^\star)^T \bar{H}^T v + v^T \dot{v}, \\
&= k_p \langle g - g^\star, \bar{H}^T v \rangle - k_p \langle \hat{g} - g^\star, \bar{H}^T v \rangle - k_v v^T \bar{H} \hat{\dot{g}} \\
&= -k_p v^T \bar{H} r_g - k_v v^T \bar{H} \hat{\dot{g}} \\
&= -k_p \dot{e}^T r_g - k_v \dot{e}^T r_{\dot{g}} - k_v \dot{e}^T \dot{g}, \\
&= -k_p \sum_i \sum_{j \in \mathcal{N}_i} \langle \dot{e}_{ij}, r_{g_{ij}} \rangle - k_v \sum_i \sum_{j \in \mathcal{N}_i} \langle \dot{e}_{ij}, r_{\dot{g}_{ij}} \rangle - k_v \sum_i \sum_{j \in \mathcal{N}_i} \langle \dot{e}_{ij}, \dot{g}_{ij} \rangle,
\end{aligned}$$

which follows from the property that $e_{ij}^T \dot{g}_{ij} = 0$ and the error definitions from (15). Using Cauchy-Schwarz inequality $|\langle a, b \rangle| \leq \|a\| \|b\|$, we bound the above equation as,

$$\dot{V}(p_i, v_i) \leq k_p \sum_i \sum_{j \in \mathcal{N}_i} \|\dot{e}_{ij}\| \|r_{g_{ij}}\| + k_v \sum_i \sum_{j \in \mathcal{N}_i} \|\dot{e}_{ij}\| \|r_{\dot{g}_{ij}}\| \quad - k_v \sum_i \sum_{j \in \mathcal{N}_i} \langle \dot{e}_{ij}, \dot{g}_{ij} \rangle.$$

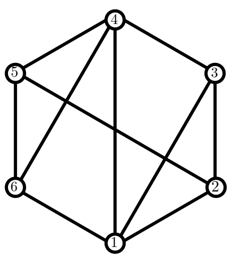
To ensure the above derivative is negative semi-definite, we will enforce, $k_p \|\dot{e}_{ij}\| \|r_{g_{ij}}\| + k_v \|\dot{e}_{ij}\| \|r_{\dot{g}_{ij}}\| \leq \sigma k_v \langle \dot{e}_{ij}, \dot{g}_{ij} \rangle$ where we again used the Cauchy-Schwarz inequality which ensures that the following derivative of the Lyapunov function,

$$\dot{V}(p_i, v_i) \leq (\sigma - 1) k_v \sum_i \sum_{j \in \mathcal{N}_i} \langle \dot{e}_{ij}, \dot{g}_{ij} \rangle,$$

is negative semi-definite for $\sigma \in (0, 1)$. Invoking the Invariance principle,[18], define $S_1 = \{(p, v) \in \mathbb{R}^{2nd} | \dot{V} = 0\}$ where $\dot{V} = 0$ implies $\dot{e}_{ij}$ and $\dot{g}_{ij}$ are orthogonal to each other. As $\dot{g}_{ij}$ and g_{ij} are orthogonal too (i.e., $\dot{g}_{ij}^T g_{ij} = 0$), then there exists a scalar γ, such that $\dot{e}_{ij} = \gamma g_{ij}$. Now from the definition of $\dot{g}_{ij}$, given by $\dot{g}_{ij} = \frac{\gamma P_{g_{ij}}}{\|e_{ij}\|} g_{ij}$, is 0 since $P_{g_{ij}}$ is the orthogonal projection of g_{ij} i.e., $P_{g_{ij}} g_{ij} = 0$. Therefore $\dot{g}_{ij} = 0$ and $g_{ij}(t)$ is constant. At this point if $g_{ij} = g_{ij}^\star$ then the proof is complete and if $g_{ij} \neq g_{ij}^\star$ then $u(t_i^k)$ in (16) evolves until $g_{ij} = g_{ij}^\star$. □

4 | SIMULATION RESULTS

In this section, we demonstrate the effectiveness of the theoretical results by simulating an example for a network of 6 agents with communication topology as shown in Figure 3. Section 4.1 demonstrates the results of Theorem 3 while Section 3.2 demonstrates the results of Theorem 4. Both examples utilize the same communication topology, desired formation, and initial conditions.

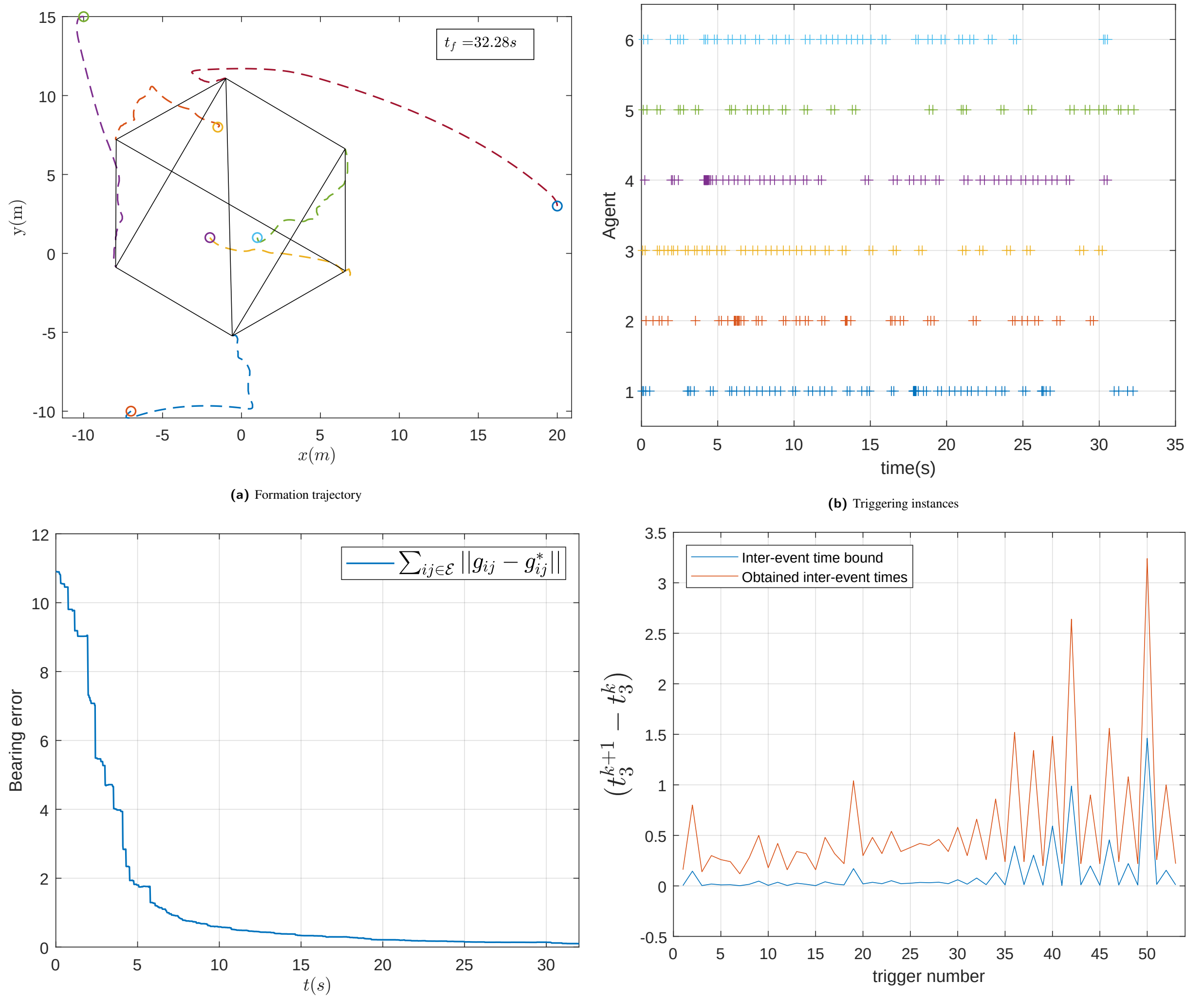


$$\begin{aligned}
g_{12}^\star &= [\sqrt{3}/2 \quad 1/2]^T & g_{61}^\star &= [\sqrt{3}/2 \quad -1/2]^T \\
g_{23}^\star &= [0 \quad 1]^T & g_{13}^\star &= [1/2 \quad \sqrt{3}/2]^T \\
g_{34}^\star &= [-\sqrt{3}/2 \quad 1/2]^T & g_{14}^\star &= [0 \quad 1]^T \\
g_{45}^\star &= [-\sqrt{3}/2 \quad -1/2]^T & g_{25}^\star &= [-\sqrt{3}/2 \quad 1/2]^T \\
g_{56}^\star &= [0 \quad -1]^T & g_{46}^\star &= [-1/2 \quad -\sqrt{3}/2]^T
\end{aligned}$$

FIGURE 3 Communication graph for the example.

4.1 | Node triggering

In node triggering, Figure 3 is simulated with position and velocity gains $k_p = 8$ and $k_v = 4$ respectively. The initial condition for the agents, denoted by circle markers, are $[-7, -10]$, $[-2, 1]$, $[1, 1]$, $[20, 4]$, $[0.5, 8]$ and $[-10, 15]$ correspondingly. The



(a) Formation trajectory

(b) Triggering instances

(c) Bearing error

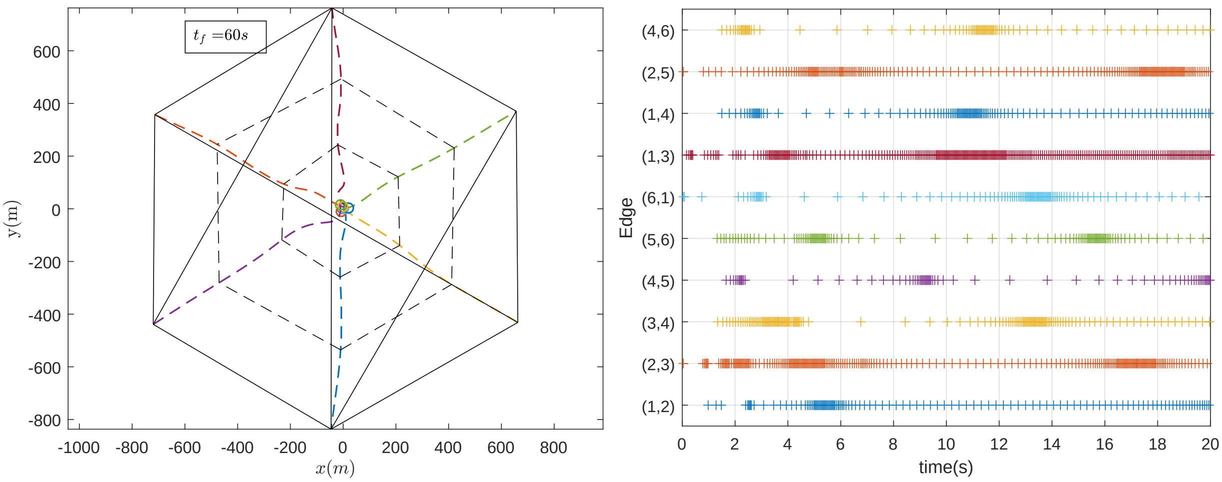
(d) Inter-event times

FIGURE 4 Results pertaining to the *node triggering* problem.

event variable σ is specified as 0.5. From Figure 4a we see that the desired formation (shown in Figure 3) is achieved with a convergence time of $t_f = 32.28$s and with a final bearing error of 0.07. It needs to be pointed that along with the control updates, the sensing is also event driven here. Figure 4b shows the instances when an event is triggered using the ETC (9), here every marker represents an instance when an agent updates its controller and acquires new bearing-state measurements. The evolution of bearing error, defined as $\sum_{(i,j)\in\mathcal{E}} \|g_{ij} - g^\star_{ij}\|$ is shown in Figure 4c and the lower bound for the inter-event time calculated for agent 3 using (11) and the actual inter-event time is shown in Figure 4d. The y-axis represents the inter-event times between two successive events k and $k+1$, while the x-axis counts the number of such events. Here we confirm the findings that actual inter-event times are strictly greater than 0.

4.2 | Edge triggering

The simulation results of Section 3.2 with $k_p = 6$ and $k_v = 8$ are presented here. For the system with dynamics (1) and control law (16) and under the action of ETC (17), as noticed in Figure 5a all 6 agents achieve the desired formation and continue moving and scaling in an increased size with a constant final velocity. The snapshots of agents at $t = 20s$ and $t = 40s$ show the formation in dotted lines, while at $t = 60s$, the formation appears in bold, as depicted in Figure 5a. . Figure 5b shows the instances when an event is triggered over a particular edge and every marker over an edge $(i, j) \in \mathcal{E}$ indicates a control update for agent i and agent j. The Figure 5b also indicates that certain edges may be more important to the system for solving the problem.



(a) Formation trajectory

(b) Triggering instances

FIGURE 5 Results pertaining to the *edge triggering* problem.

5 | CONCLUSION

In this work, we presented distributed bearing-based event-triggered schemes to achieve formation stabilization for agents with double-integrator dynamics. For node triggering, we used additional information in the form of self-velocity to arrive at an ETC that uses collective information from all its neighbors to render the final formation stationary and ensured the inter-event time is non-zero. In edge triggering, we demonstrated how each edge can have its own ETC and drive the control updates of the agent. We also presented a simulation example to establish the effectiveness of these results and showed that event-triggered control can substantially decrease the amount of processing power by sensing and updating controller at event times rather than traditional time-triggered updates. In future work we plan to consider additional performance metrics and study the robustness of these ETC solutions to model uncertainties and disturbances.

References

1. Ren W, Beard RW. Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Transactions on Automatic Control* 2005; 50(5): 655–661.

2. Fax JA, Murray RM. Information flow and cooperative control of vehicle formations. *IEEE Transactions on Automatic Control* 2004; 49(9): 1465–1476.

3. Oh KK, Park MC, Ahn HS. A survey of multi-agent formation control. *Automatica* 2015; 53: 424–440.

4. Zhao S, Zelazo D. Bearing rigidity and almost global bearing-only formation stabilization. *IEEE Transactions on Automatic Control* 2016; 61(5): 1255–1268.

5. Zhao S, Zelazo D. Translational and scaling formation maneuver control via a bearing-based approach. *IEEE Transactions on Control of Network Systems* 2015; 4(3): 429–438.

6. Zhao S, Li Z, Ding Z. Bearing-Only Formation Tracking Control of Multiagent Systems. *IEEE Transactions on Automatic Control* 2019; 64(11): 4541-4554.

7. Dimarogonas DV, Frazzoli E, Johansson KH. Distributed event-triggered control for multi-agent systems. *IEEE Transactions on Automatic Control* 2011; 57(5): 1291–1297.

8. Sewlia M, Zelazo D. Distributed Event-Based Control for Second-Order Multi-Agent Systems. In: *27th Mediterranean Conference on Control and Automation (MED)*IEEE. ; 2019: 310–315.

9. Nowzari C, Garcia E, Cortés J. Event-triggered communication and control of networked systems for multi-agent consensus. *Automatica* 2019; 105: 1–27.

10. Yu Y, Zeng Z, Li Z, Wang X, Shen L. Event-triggered encirclement control of multi-agent systems with bearing rigidity. *Science China Information Sciences* 2017; 60(11): 110203.

11. Ge X, Han QL. Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism. *IEEE Transactions on Industrial Electronics* 2017; 64(10): 8118–8127.

12. Zhang L, Li X, Yan J, Guan X. Event-triggered multitarget formation control for multiagent systems. *Mathematical Problems in Engineering* 2017; 2017.

13. Sun Z, Liu Q, Yu C, Anderson BD. Generalized controllers for rigid formation stabilization with application to event-based controller design. In: *2015 European Control Conference (ECC)*IEEE. ; 2015: 217–222.

14. Cheng B, Wu Z, Li Z. Distributed edge-based event-triggered formation control. *IEEE transactions on cybernetics* 2019.

15. Mishra R, Ishii H. Event-triggered control for discrete-time multi-agent average consensus. *International Journal of Robust Nonlinear Control* 2021. doi: 10.1002/rnc.5815

16. Yu P, Ding L, Liu ZW, Guan ZH. Leader–follower flocking based on distributed event-triggered hybrid control. *International Journal of Robust and Nonlinear Control* 2016; 26(1): 143–153.

17. Li Z, Tnunay H, Zhao S, Meng W, Xie SQ, Ding Z. Bearing-Only Formation Control With Prespecified Convergence Time. *IEEE Transactions on Cybernetics* 2022; 52(1): 620-629. doi: 10.1109/TCYB.2020.2980963

18. Khalil HK, Grizzle J. *Nonlinear systems*. 3. Prentice hall Upper Saddle River, NJ . 2002.

19. Tabuada P. Event-Triggered Real-Time Scheduling of Stabilizing Control Tasks. *IEEE Transactions on Automatic Control* 2007; 52(9): 1680-1685.

How to cite this article: M. Sewlia, and D. Zelazo (2021), Bearing-Based Formation Stabilization Using Event-Triggered Control